

# DUALITY FOR CONSTRAINED ROBUST SUM OPTIMIZATION PROBLEMS

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## Abstract

Given an infinite family of extended real-valued functions  $f_i$ ,  $i \in I$ , and a family  $\mathcal{H}$  of nonempty finite subsets of  $I$ , the  $\mathcal{H}$ -partial robust sum of  $f_i$ ,  $i \in I$ , is the supremum, for  $J \in \mathcal{H}$ , of the finite sums  $\sum_{j \in J} f_j$ . These infinite sums arise in a natural way in location problems as well as in functional approximation problems, and include as particular cases the well-known sup function and the so-called robust sum function, corresponding to the set  $\mathcal{H}$  of all nonempty finite subsets of  $I$ , whose unconstrained minimization was analyzed in previous papers of three of the authors [DOI: 10.1007/s11228-019-00515-2 and DOI: 10.1007/s00245-019-09596-9]. In this paper, we provide ordinary and stable zero duality gap and strong duality theorems for the minimization of a given  $\mathcal{H}$ -partial robust sum under constraints, as well as closedness and convex criteria for the formulas on the subdifferential of the sup-function.

Key words: partial robust sums, robust sum optimization problems, stable zero duality gap