On a problem for nonlinear diffusion equation with conformable time derivative

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Abstract

In this work, we study a nonlinear diffusion equation with conformable derivative: $\mathfrak{D}_t^{(\alpha)}u - \Delta u = \mathcal{L}(x,t;u(x,t))$, where $0 < \alpha < 1,\ (x,t) \in \Omega \times (0,T)$. We consider both of the problems:

- Initial value problem: the solution contains the integral $I = \int_0^t \tau^\gamma \mathrm{d}\tau$, (critical as $\gamma \leq -1$).
- Final value problem: not well-posed (if the solution exists it does not depend continuously on the given data).

For the initial value problem, the lack of convergence of the integral I, for $\gamma \leq -1$. The existence of the solution is represented. For final value problem, the Hadamard-instability occurs, we propose two regularization methods to solve the nonlinear problem in case the source term is a Lipschitz function. The results of existence, uniqueness and stability of the regularized problem are obtained. We also develop some new techniques on functional analysis to propose regularity estimates of regularized solution.