ON THE EXISTENCE OF MULTIPLE SOLUTIONS FOR A CLASS OF ELLIPTIC EQUATIONS INVOLVING $p(x)$-LAPLACIAN.

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Abstract. In this talk we study the existence of multiple solutions for the following problem:

$$\begin{cases} -\operatorname{div}\left(w(x)|\nabla u|^{p(x)-2}\nabla u\right) = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with Lipschitz boundary $\partial\Omega$, $p : \overline{\Omega} \to (1,\infty)$ is a continuous function, w is a weighted function in Ω and f is a Carathéodory function. Firstly, using a critical point theorem of Kajikiya (2005) together with an a-priori bound of solutions we obtain a sequence of solutions tending to zero under a p$(\cdot)$-sublinear growth condition of nonlinearity. Secondly, for a p$(\cdot)$-superlinear growth condition, we obtain a sequence of solutions tending to infinity when nonlinear term satisfies a condition called Ambrosetti-Rabinowitz type using genus theory.

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