

# THE GERSTENHABER-SCHACK COMPLEX FOR PRESTACKS

DINH VAN HOANG

ABSTRACT. Building on the work of Gerstenhaber and Schack for presheaves of algebras, we define a Gerstenhaber-Schack complex  $\mathbf{C}_{\text{GS}}^\bullet(\mathcal{A})$  for an arbitrary prestack  $\mathcal{A}$ , that is a pseudofunctor taking values in linear categories over a commutative ground ring. In the general case, the differential is no longer simply the sum of Hochschild and simplicial contributions as in the presheaf case, but contains additional higher components as well. If  $\tilde{\mathcal{A}}$  denotes the Grothendieck construction of  $\mathcal{A}$ , which is a  $\mathcal{U}$ -graded category, we explicitly construct inverse quasi-isomorphisms between  $\mathbf{C}_{\text{GS}}^\bullet(\mathcal{A})$  and the Hochschild complex  $\mathbf{C}_{\mathcal{U}}(\tilde{\mathcal{A}})$ . As the Homotopy Transfer Theorem applies to our construction, one can transfer the dg Lie structure present on the Hochschild complex in order to obtain an  $L_\infty$ -structure on  $\mathbf{C}_{\text{GS}}^\bullet(\mathcal{A})$ , which controls the higher deformation theory of the prestack  $\mathcal{A}$ . (This is a joint work with Wendy Lowen)