ON THE MEASURE-VALUED SOLUTION OF A SPECTRAL FRACTIONAL FORWARD-BACKWARD EQUATION

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In this talk, we prove existence of a measure-valued solution of a forward-backward equation with spectral fractional Laplacian. More precisely, we study the following fractional parabolic problem for $0 < \alpha \leq 1$ with Dirichlet boundary condition:

$$(P) \qquad \begin{cases} u_t = -(-\Delta)^\alpha \varphi(u) & \text{in } \Omega \times (0,T) =: Q_T \\ u = 0 & \text{in } \partial\Omega \times (0,T) \\ u = u_0 & \text{in } \Omega \times \{0\}, \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$ and φ is a non-monotone function which satisfies the following assumption
$(H1)$

$$\varphi \in C^2(\mathbb{R}),\ \varphi(0) = 0,\ \ \varphi'(s) > 0 \text{ for } |s| > s_0, \text{ for some } s_0 > 0.$$

The operator $A_\alpha := (-\Delta)^\alpha$ is defined for any $u \in L^2(\Omega)$ and $u = \sum_{k=1}^\infty u_k e_k$, by

$$A_\alpha u = \sum_{k=1}^\infty \lambda_k^\alpha u_k e_k$$

where $\{e_k, \lambda_k\}_{k=1}^\infty$ denote an orthonormal basis of $L^2(\Omega)$ consisting of eigenfunctions of $-\Delta$ in Ω with homogeneous Dirichlet boundary conditions and their corresponding eigenvalues. Our main method is to investigate the vanishing viscosity of solutions (u_ϵ) of problem:

$$(P_\epsilon) \qquad \begin{cases} u_t = -(-\Delta)^\alpha \left(\varphi(u) + \epsilon u_t\right) & \text{in } \Omega \times (0,T) =: Q_T \\ \varphi(u) + \epsilon u_t = 0 & \text{in } \partial\Omega \times (0,T) \\ u = u_0 & \text{in } \Omega \times \{0\}, \end{cases}$$

References

[1] T. T. L. Bui, *On the well-posedness of a spectral fractional forward-backward pseudo-parabolic equation*, preprint.